Sample answer for review problems

Working example 1
****The mathematical notation for Chi-square tests was not covered in the text. So, writing the hypotheses in words is fine.**

H_o: There are equal numbers of people who got married in the 3 age groups.
H_1: There are different numbers of people who got married in the 3 age groups.

Expected value for the three faculty members = $\frac{\sum f_o}{k} = \frac{10+24+7}{3} = \frac{41}{3} = 13.6667$

	Before 23	**23-27**	**After 27**
Observed	10	24	7
Expected	13.6667	13.6667	13.6667

$$\chi^2_{obs} = \sum \frac{(O-E)^2}{E} = \frac{(10-13.6667)^2}{13.6667} + \frac{(24-13.6667)^2}{13.6667} + \frac{(7-13.6667)^2}{13.6667}$$
$$= \frac{13.4447}{13.6667} + \frac{106.7771}{13.6667} + \frac{44.4449}{13.6667} = 12.049$$

Degrees of freedom = C – 1 = 3-1 = 2 $\alpha = .05$

From table on page 736, χ^2_{crit} (2, .05) = 5.99

Because $\chi^2_{obs} > \chi^2_{crit}(2,.05)$, we reject Ho. Thus, there are different numbers of people who got married in the three age groups. Notably, we know there are more people in the middle group, but we are not sure if the numbers of people differ in group 1 (Before 23) and group 3 (After 27).

Working Example 2

H_o: The age of first marriage does not predict happiness after marriage.
H_1: The age of first marriage predicts happiness after marriage.

<u>Age of first marriage</u>

	Before 23	23-27	After 27	**Total**
Happier before marriage	16 (15)	18 (18)	8 (9)	42
Happier after marriage	9 (10)	12 (12)	7 (6)	28
Total	25	30	15	70

Note: Expected values in parentheses.

Expected value = $E_{ij} = \frac{R_i C_j}{N}$ For example, $E_{11} = \frac{42 \times 25}{70} = 15$

$$\chi^2_{obs} = \sum \frac{(O-E)^2}{E} = \frac{(16-15)^2}{15} + \frac{(18-18)^2}{18} + \frac{(8-9)^2}{9} + \frac{(9-10)^2}{10} + \frac{(12-12)^2}{12} + \frac{(7-6)^2}{6}$$
$$= 0.06667 + 0 + 0.1111 + 0.1 + 0 + 0.16667 \approx 0.4444$$

Degrees of freedom = (R-1)(C-1) = (1)(2) = 2 $\alpha = .05$
From table on page 736, χ^2_{crit} (2, .05) = 5.99

Because $\chi^2_{obs} > \chi^2_{crit}(2,.05)$, we fail to reject Ho. Thus, age of first marriage does not predict happiness after marriage.

The measure of association I would choose for this problem is Cramér's Phi because the dimension of this Chi-sq table is larger than 2x2.

Working Example 3

Sample	Population
$\bar{x} = 24$	$\mu = 28$
$s = ??$	$\sigma = 4$
$n = 70$	

H_o: The sample of friends I tested is from the Canadian women population with the age of first marriage at 28. ($\mu = 28$)
H_1: The sample of friends I tested is NOT from the Canadian women population with the age of first marriage at 28. ($\mu \neq 28$)

Because population σ is known, we use z-formula

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{70}} = 0.4781 \qquad z_{obs} = \frac{\bar{X} - \mu}{\sigma_{\bar{x}}} = \frac{24-28}{0.4781} = -8.3666$$

$Z_{crit} = \pm 1.96$ with $\alpha = .05$

Because $p(z > z_{obs}) < .05$, we reject H_o. There is evidence that the sample of friends I tested is not from the Canadian women population based on their age of first marriage.

Working Example 4

Sample	Population
$\bar{x} = 24$	$\mu = 26.8$
$s = 2$	$\sigma = ??$
$n = 70$	

H_o: The sample of friends I tested is from the women population in developed countries with the age of first marriage at 26.8 ($\mu = 26.8$)

H_1: The sample of friends I tested is NOT from the women population in developed countries with the age of first marriage at 26.8 ($\mu \neq 26.8$).

Because population σ is unknown, we use t-formula

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{2}{\sqrt{70}} = 0.2390 \qquad t_{obs} = \frac{\bar{X} - \mu}{s_{\bar{x}}} = \frac{24 - 26.8}{0.2390} = -11.7155$$

$t_{crit} = t\ (69) \approx t\ (50) = 2.009$ with $\alpha = .05$

Because $t_{obs} > t_{crit}$, we reject H_o. There is evidence that the sample of friends I tested is not from the women population in developed countries based on their age of first marriage.

Working Example 5

Get the difference scores by subtracting the husbands' ratings from the wives' for each couple:

Wife:	8	6	8	4	6	6	9	4	1	3	6	8
Husband:	6	8	4	2	5	3	8	7	2	4	5	7
Difference:	2	-2	4	2	1	3	1	-3	-1	-1	1	1

H_o: There is no significant difference between the wives' and the husbands' ratings of happiness. ($\mu_D = 0$)

H_1: There is no significant difference between the wives' and the husbands' ratings of happiness ($\mu_D \neq 0$)

Sample info: $\bar{D} = 8 \div 12 = .6667, s = 2.0597$, n = 12

Because the difference scores are provided, we will carry out the t-test for matched samples.

$$t_{obs} = \frac{\bar{D} - 0}{s_{\bar{D}}} = \frac{.6667 - 0}{2.0597 / \sqrt{12}} = \frac{.6667}{0.5946} = 1.121$$

df = 12-1 = 11 $\alpha = .05$ $t_{crit} = \pm 2.201$

Because $t_{obs} < |t_{crit}|$, we fail to reject H_o. Thus, there is evidence that the husbands' and wives' happiness ratings are consistent.

Working Example 6

$\overline{X}_{23-27} = 6.59$ $\overline{X}_{after27} = 7.23$

$s_{23-27} = 2.61$ $s_{after27} = 3.2$

$s^2_{23-27} = 6.8121$ $s^2_{after27} = 10.24$

$n_{23-27} = 12$ $n_{after27} = 12$

a) $$s_p^2 = \frac{(n_{after27}-1)s^2_{after27} + (n_{23-27}-1)s^2_{23-27}}{n_{after27}+n_{23-27}-2} = \frac{(12-1)10.24+(12-1)6.8121}{12+12-2} = 8.52605$$

b) H_o: There is no difference in happiness between the groups that first got married at the ages of 23-27 and after the age of 27 ($\mu_1 - \mu_2 = 0$)

H_1: There is no difference in happiness between the groups that first got married at the ages of 23-27 and after the age of 27 ($\mu_1 - \mu_2 \neq 0$)

$$t_{obs} = \frac{\overline{X}_{after27} - \overline{X}_{23-27}}{s_{\bar{x}_{after27}-\bar{x}_{23-27}}} = \frac{\overline{X}_{after27} - \overline{X}_{23-27}}{\sqrt{\frac{s_p^2}{n_{after27}} + \frac{s_p^2}{n_{23-27}}}} = \frac{7.23-6.59}{\sqrt{\frac{8.52605}{12}+\frac{8.52605}{12}}} = \frac{0.64}{1.19206} = 0.5369$$

df = 12+12-2 = 22 $\alpha = .05$ $t_{crit} \approx \pm 2.074$

Because $t_{obs} < |t_{crit}|$, we fail to reject H_o. There is not enough evidence showing that married couples are happier if they first got married between the age of 23-27 compared to getting married after the age of 27.

c) Cohen's d = $$\frac{\overline{X}_{after27} - \overline{X}_{23-27}}{s_p} = \frac{7.23-6.59}{2.9199} = 0.2192$$

d) There is a possible problem of heterogeneity of variances. Possible solution is to adopt the Welch-Satterthwaite method.

Working Example 7

To calculate correlation between husbands' and wives' happiness:

Step 1. **Find covariance**.

$\sum husband = 61$ $\sum wife = 69$

$\sum husband - wife = (X_1 \cdot Y_1) + (X_2 \cdot Y_2) + ... + (X_{10} \cdot Y_{10}) = 384$ N = 12

$$\text{cov} = \frac{384 - \frac{(61)(69)}{12}}{12-1} = \frac{384-350.75}{11} = 3.0227$$

Step 2. Find sample standard deviations for husbands and wives.

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{N}}{N-1} \qquad \sum husband_2^2 = 361 \qquad \sum wife^2 = 459$$

$$s_{husband} = \sqrt{\frac{361 - \frac{(61)^2}{12}}{11}} = 2.15146 \qquad s_{wife} = \sqrt{\frac{459 - \frac{(69)^2}{12}}{11}} = 2.3789$$

Step 3. Calculate r.

$$r = \frac{\text{cov}}{s_{husband}s_{wife}} = \frac{3.0227}{(2.15146)(2.3789)} = \frac{3.0227}{5.1181} = 0.5906 \approx 0.59$$

$$r_{adj} = \sqrt{1 - \frac{(1-r^2)(N-1)}{N-2}} = \sqrt{1 - \frac{(1-0.59^2)(12-1)}{12-2}} = 0.5319$$

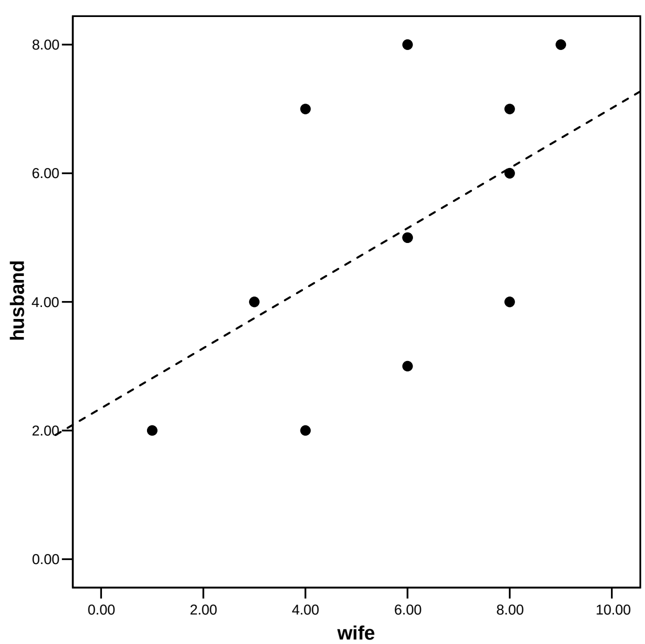
Ho: The population correlation between husbands and wives' happiness is not significantly different from 0. ($\rho = 0$)
H_1: The population correlation between husbands and wives' happiness is significantly different from 0. ($\rho \neq 0$)

$$t_{obs} = \frac{r\sqrt{N-2}}{\sqrt{1-r^2}} = \frac{(0.59)\sqrt{12-2}}{\sqrt{1-(.59)^2}} = \frac{(.59)(3.16228)}{.8074} = 2.3108$$

df = N – 2 = 12 – 2 = 10 $\alpha = .05$ $t_{crit} = \pm 2.201$

Since $t_{obs} > t_{crit}$, we reject Ho. Therefore, there is evidence showing that the correlation between husbands and wives' happiness is significantly different from zero.

Working Example 8



$$\hat{Y} = bX + a$$

$$b = \frac{\text{cov}}{s_{wife}^2} = \frac{3.0227}{2.3789^2} = \frac{3.0227}{5.6592} = 0.5341$$

$$a = \overline{Y} - b\overline{X} = 5.0833 - (0.5341)(5.75) = 2.0122$$

$$\therefore \hat{Y} = 0.5341X_1 + 2.0122$$

Standard error of estimate

$$s_{Y.X} = s_Y\sqrt{(1-r^2)\frac{N-1}{N-2}}$$

$$s_{Y \cdot X} = (2.3789)\sqrt{(1-0.59^2)\frac{12-1}{12-2}} = (2.3789)\sqrt{(0.6519)(1.1)} = 2.0145$$